Surname	Centre Number	Candidate Number
Other Names		2

GCE A level

1325/01

S15-1325-01

PHYSICS – PH5
Electromagnetism, Nuclei & Options

A.M. THURSDAY, 18 June 2015

1 hour 45 minutes

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010001

ADDITIONAL MATERIALS

In addition to this paper, you will require a calculator, a **Case Study Booklet** and a **Data Booklet**.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen. Do not use pencil or gel pen. Do not use correction fluid.

Write your name, centre number and candidate number in the spaces at the top of this page.

Write your answers in the spaces provided in this booklet. If you run out of space, use the continuation pages at the back of the booklet, taking care to number the question(s) correctly.

INFORMATION FOR CANDIDATES

This paper is in 3 sections, **A**, **B**, and **C**.

Section A: 60 marks. Answer **all** questions. You are advised to spend about 1 hour on this section.

Section B: 20 marks. The Case Study. Answer **all** questions. You are advised to spend about 20 minutes on this section.

Section C: Options; 20 marks. Answer **one option only.** You are advised to spend about 20 minutes on this section.

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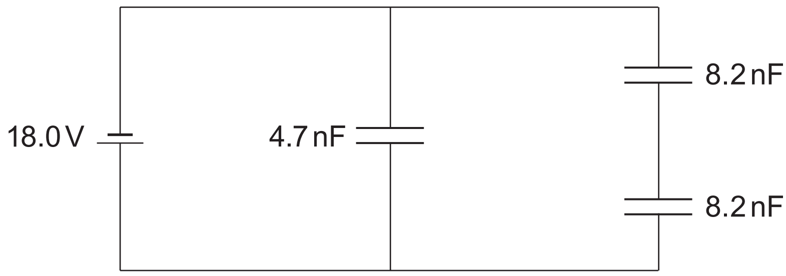
Examiner only

SECTION A

Answer **all** *questions.*

1. This question is about capacitors in the following circuit.



(a) (i) Calculate the charge stored by each of the capacitors. [2]

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(ii) Calculate the total energy stored by all three capacitors. [2]

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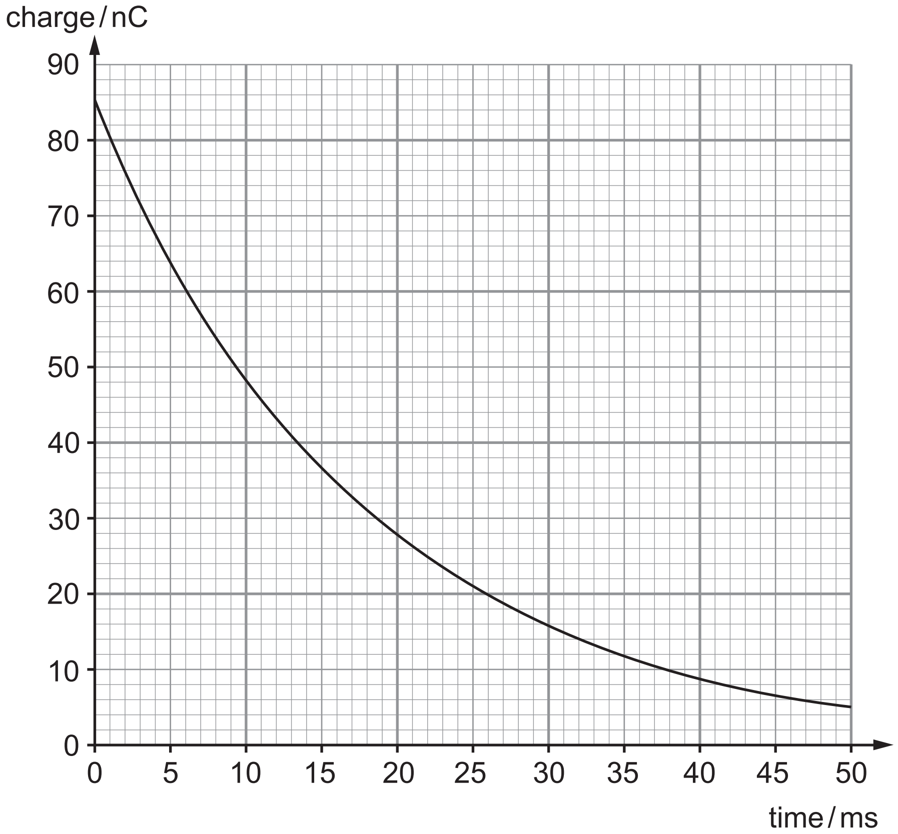
(b) The 4.7 nF capacitor is discharged through an unknown resistor and the following graph is obtained.



(i) Calculate the resistance of the resistor. [3]

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(ii) Calculate the current when t = 0 s. [2]

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Turn over.

Examiner only

(iii) By estimating the time taken for the capacitor to lose 90% of its charge or otherwise, calculate the time taken for the capacitor to lose 99% of its charge. [2]

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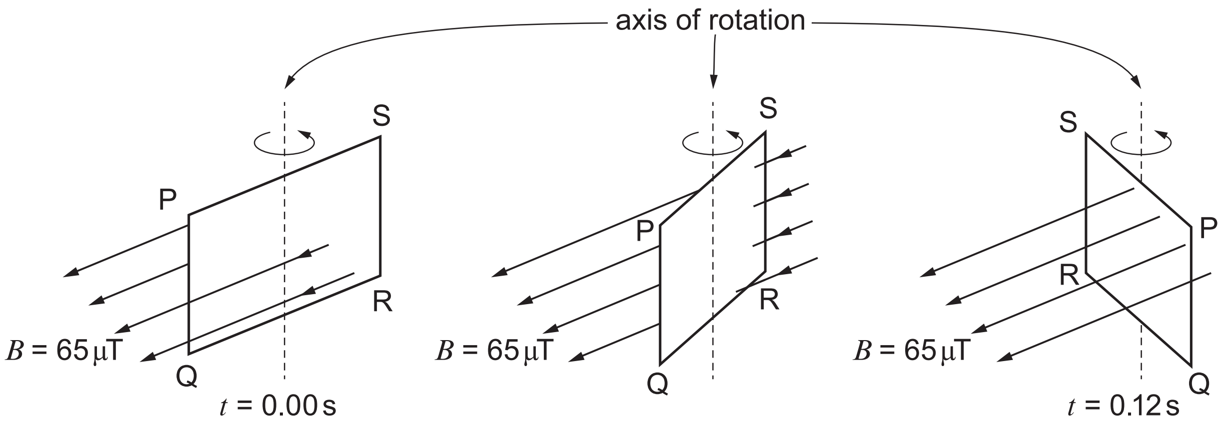
2. A large square loop has sides of length 0.815 m and is rotated through 90° in a uniform magnetic field of 65 μT. The diagrams show the same square loop at different times.



(a) Determine the magnetic flux through the square loop: [3]

(i) when t = 0.00 s (sides QR and SP are parallel to the B-field);

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(ii) and when t = 0.12 s (PQ, QR, RS and SP are perpendicular to the B-field).

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(b) The square loop is made of copper. Explain why there is a current in the loop as it is rotated. [2]

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Examiner only

(c) Explain how Lenz's law will give the direction of the forces acting on the sides PQ and RS as the square loop is rotated. [2]

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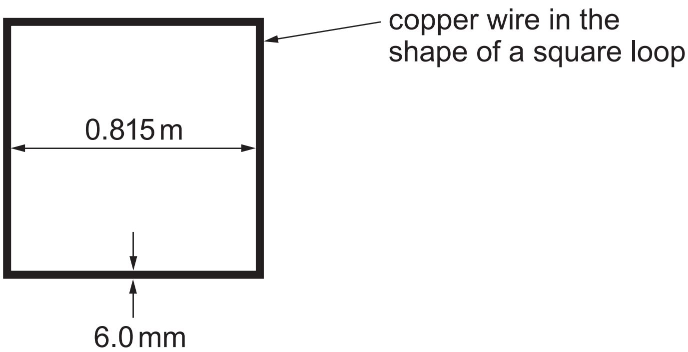
(d) The copper wire from which the square loop is made has a circular cross-section of diameter 6.0 mm. The resistivity of copper is $1.67 \times 10^{-8}\,\Omega\,\text{m}$. Calculate the **mean** current flowing through the square loop as it is rotated between $t = 0.00\,\text{s}$ and $t = 0.12\,\text{s}$. [5]



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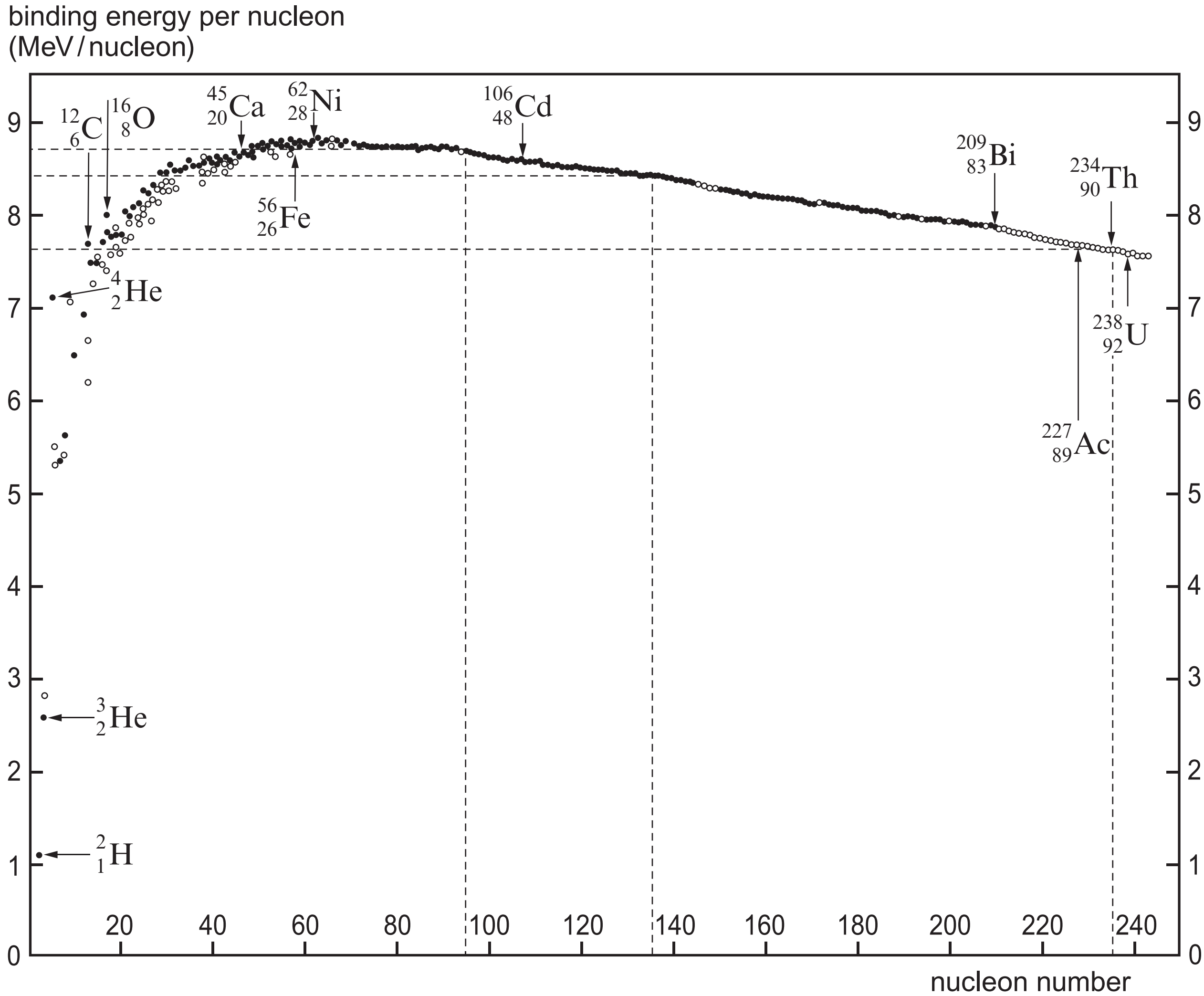
3. *(a)* Some nuclei undergo fusion while others undergo fission. Both processes can result in the release of energy. Discuss these processes in terms of energy and stability. The binding energy per nucleon graph is provided to assist your answer. [4]



Examiner only

(b) Use data from the graph to estimate the energy released in the reaction. [2]

$$^{1}_{0}\mathrm{n} + {}^{1}_{1}\mathrm{p} \longrightarrow {}^{2}_{1}\mathrm{H}$$

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(c) Use the graph to estimate the energy released in the following reaction. (Hint: use the binding energies on both sides of the reaction equation.) [4]

$$^{1}_{0}\mathrm{n} + {}^{235}_{92}\mathrm{U} \longrightarrow {}^{137}_{55}\mathrm{Cs} + {}^{96}_{37}\mathrm{Rb} + 3\,{}^{1}_{0}\mathrm{n}$$

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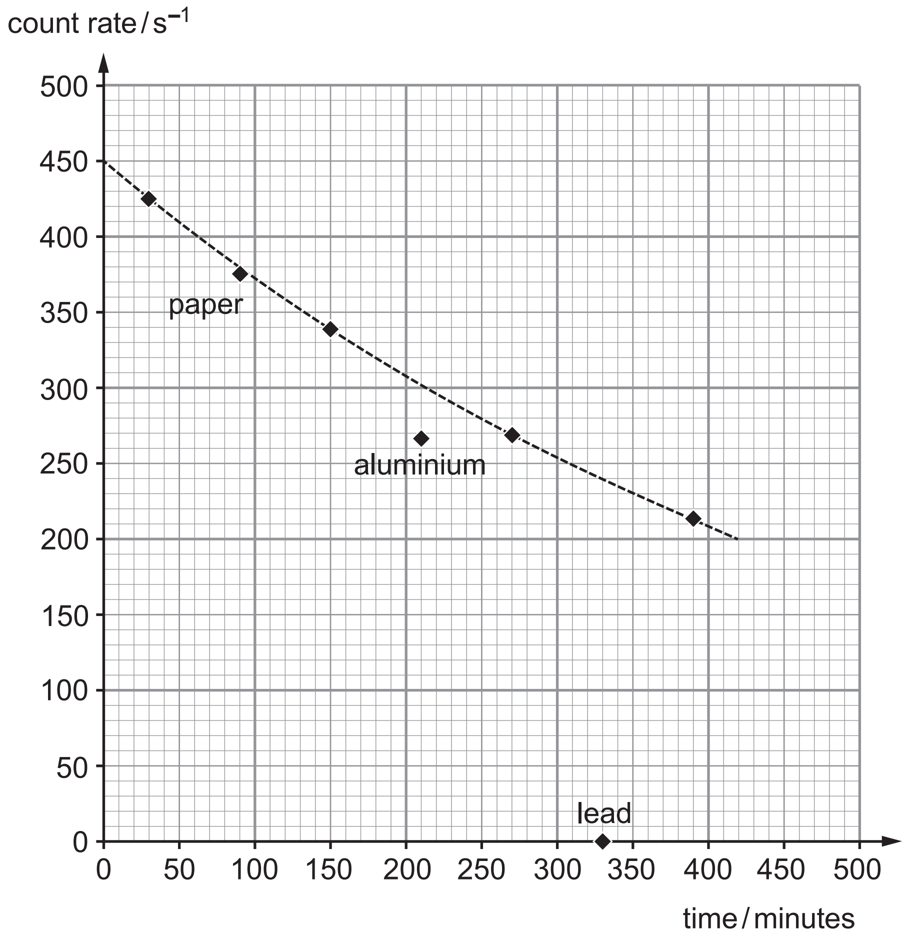
Examiner only

4. Technetium-99 emits **only** γ (gamma) radiation. An experiment was carried out to show this. Various absorbers were placed between the source and detector at the times shown in the table below and the mean count rate was obtained.

Absorber	Time from the start of the experiment/min	Count rate/s^{-1}
none	30	425
1 sheet (0.1 mm) of paper	90	374
none	150	338
3 mm of aluminium	210	267
none	270	268
10 cm of lead	330	1
none	390	213

These results were plotted on a graph. The decay curve of the technetium-99 itself is plotted as a dotted line which shows the activity dropping continuously as the experiment proceeded.



Examiner only

(a) Determine the half-life of technetium-99. [1]

(b) Explain why the results are consistent with technetium-99 only emitting γ radiation. [3]

(c) The detector only detects 0.6% of the γ radiation emitted by the source. Use the graph and the half-life of technetium-99 to calculate the initial mass of technetium-99 (the mass of a technetium-99 atom is 99 u). [4]

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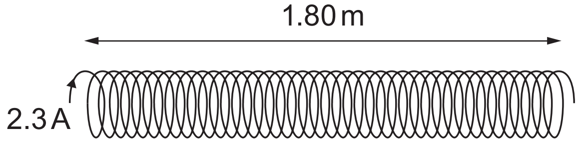
5. *(a)* A long thin solenoid carries a current of 2.3 A, has 12 000 turns and a length of 1.80 m. Calculate the magnetic field strength (B) in the centre of the long solenoid. [2]



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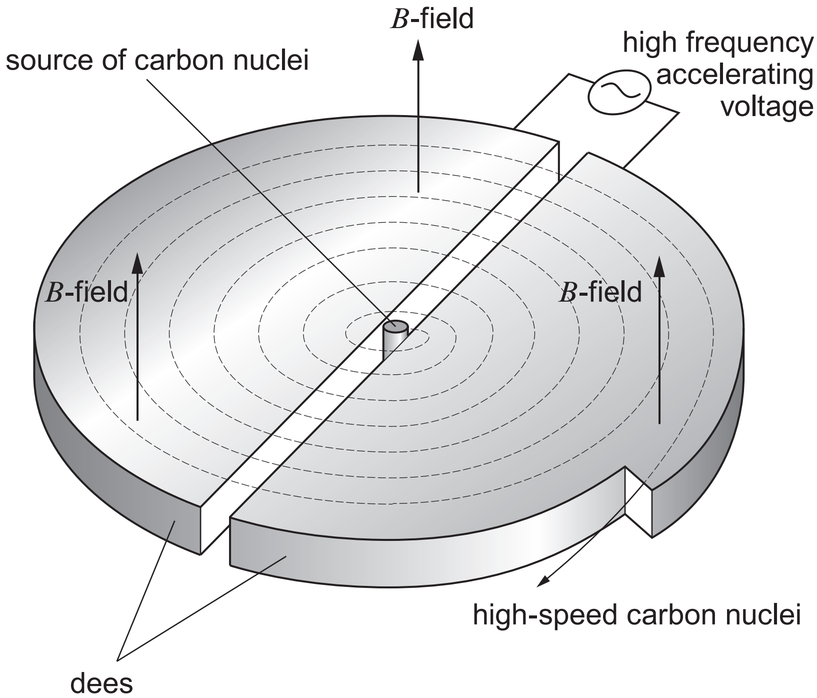
(b) In a cyclotron a uniform magnetic field (B) provides a centripetal force while an electric field accelerates the charged particles as they cross between the dees. The resulting motion is a spiral as shown.



Examiner only

(i) By equating the centripetal force to the magnetic force, show that the frequency of the a.c. supply is given by: [3]

$$f = \frac{Bq}{2\pi m}$$

where q is the charge and m is the mass of the particle.

(ii) Calculate the cyclotron frequency for a carbon nucleus with $q = 6e$ and mass $m = 12\,\text{u}$ in a strong B-field of 3.3 T. [2]

(iii) Calculate the final speed of a carbon nucleus after it has completed 12 'orbits' of the cyclotron and the potential difference between the dees is 14.5 kV (assume that the carbon nucleus starts from rest). [4]

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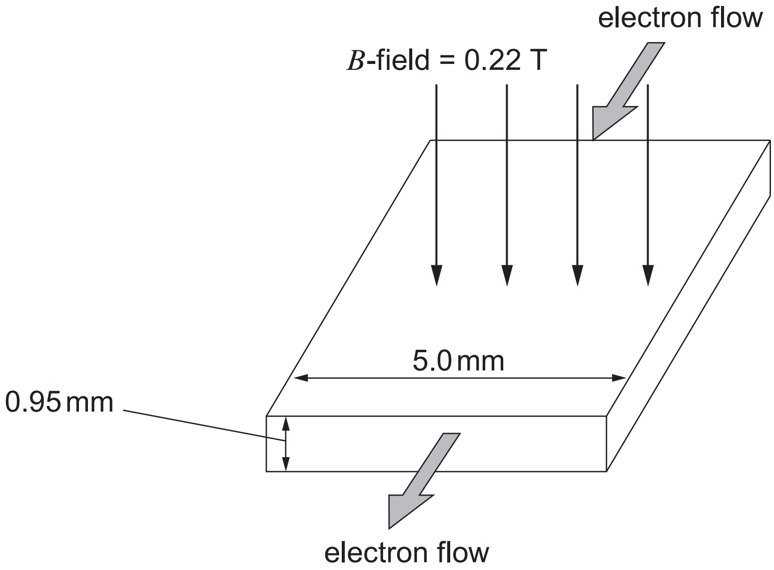
Examiner only

6. Electrons flow through a semiconductor slice which is used as a Hall probe.



(a) Indicate on the above diagram:

(i) the face of the slice that becomes positively charged; [1]

(ii) a voltmeter suitably connected to measure the Hall voltage. [1]

(b) Calculate the Hall voltage if the drift velocity of the free electrons is $5.7 \times 10^{-3}\,\text{m}\,\text{s}^{-1}$. [3]

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(c) As electrons move through the slice, explain why no work is done on them by the Hall voltage. [1]

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Examiner only

(d) The concentration of free electrons in the semiconductor slice is $7.0 \times 10^{22}\ m^{-3}$. Calculate the current in the slice. [2]

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Examiner only

SECTION B

Answer **all** *questions.*

The questions refer to the case study.
Direct quotes from the original passage will not be awarded marks.

7. *(a)* Give two reasons why only a **small** fraction of the work done in compressing the gas is transferred to gravitational potential energy of the football (paragraphs 3 & 4). Note that losses due to heat and sound are negligible. [2]

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(b) Use the values u = 20 m s^{-1}, m_0 = 1.5 kg and $\frac{\Delta m}{\Delta t}$ = 5.9 kg s^{-1} to calculate the speed of the rocket after 0.175 s (paragraph 11 and equation 2). [2]

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(c) Check that the units (or dimensions) of equation 4 are correct.

$$\frac{\Delta m}{\Delta t} = \pi r^2 \rho u$$

[2]

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Examiner only

(d) Calculate the exhaust speed of water from the rocket assuming a rate of change of mass of 9.5 kg s^{-1} and the radius of the bottle neck is 1.1 cm using equation 4 (density of water = 1 000 kg m^{-3}). [2]

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(e) Using your own words explain why 'the actual rocket does not keep up with its theoretical counterpart' (paragraphs 16-19 and equation 6). [3]

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(f) Calculate the initial exhaust speed of water leaving a bottle pumped to a pressure of 7.8×10^5 Pa (the outside atmospheric pressure is 1.0×10^5 Pa) using equation 6 (density of water = 1 000 kg m^{-3}). [2]

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Examiner only

(g) The table below refers to the terms on the right hand side of equation 8.

$$F_{res} = \pi r^2 \rho u^2 - mg - 0.0107v^2$$

Complete the table, the first row has been completed for you (paragraphs 20-22). [3]

Term	Description	During the first 0.2 s, this term		
		increases	remains constant	decreases
$\pi r^2 \rho u^2$	*Thrust force from exhaust water*			✓
mg				
$0.0107v^2$				

(h) Show that the first term ($\pi r^2 \rho u^2$) in equation 8 can be written as $2(p - p_{atm}) \times A_{neck}$ where A_{neck} is the cross-sectional area of the bottle opening (see equation 5 or 6). [2]

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(i) In practice, using Boyle's law is inappropriate because the gas cools as it expands.

(i) Explain why little or no heat flows when the gas in the bottle expands. [1]

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(ii) Use the first law of thermodynamics to explain why the temperature of the gas decreases. [1]

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SECTION C: OPTIONAL TOPICS

Option A: **Further Electromagnetism and Alternating Currents** ☐

Option B: **Revolutions in Physics – The Newtonian Revolution** ☐

Option C: **Materials** ☐

Option D: **Biological Measurement and Medical Imaging** ☐

Option E: **Energy Matters** ☐

Answer the question on **one topic only.**

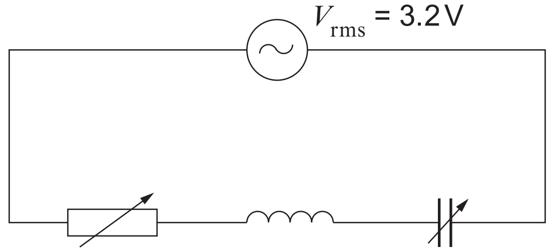
Place a tick (✓) in one of the boxes above, to show which topic you are answering.

You are advised to spend about 20 minutes on this section.

Examiner only

Option A: Further Electromagnetism and Alternating Currents

8. *(a)* In the following circuit, both the Q factor and the resonance frequency can be varied by varying the capacitance and the resistance. However, the inductance remains constant.



(i) Show that the maximum and minimum resonance frequencies are approximately 8 800 Hz and 3 600 Hz respectively. [3]

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(ii) Calculate the maximum Q factor and the minimum Q factor. [4]

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Examiner only

(b) When R = 35 Ω and C = 15 nF, the frequency of the supply is adjusted to twice the resonance frequency (approximately 17 600 Hz). Calculate the rms current. [3]

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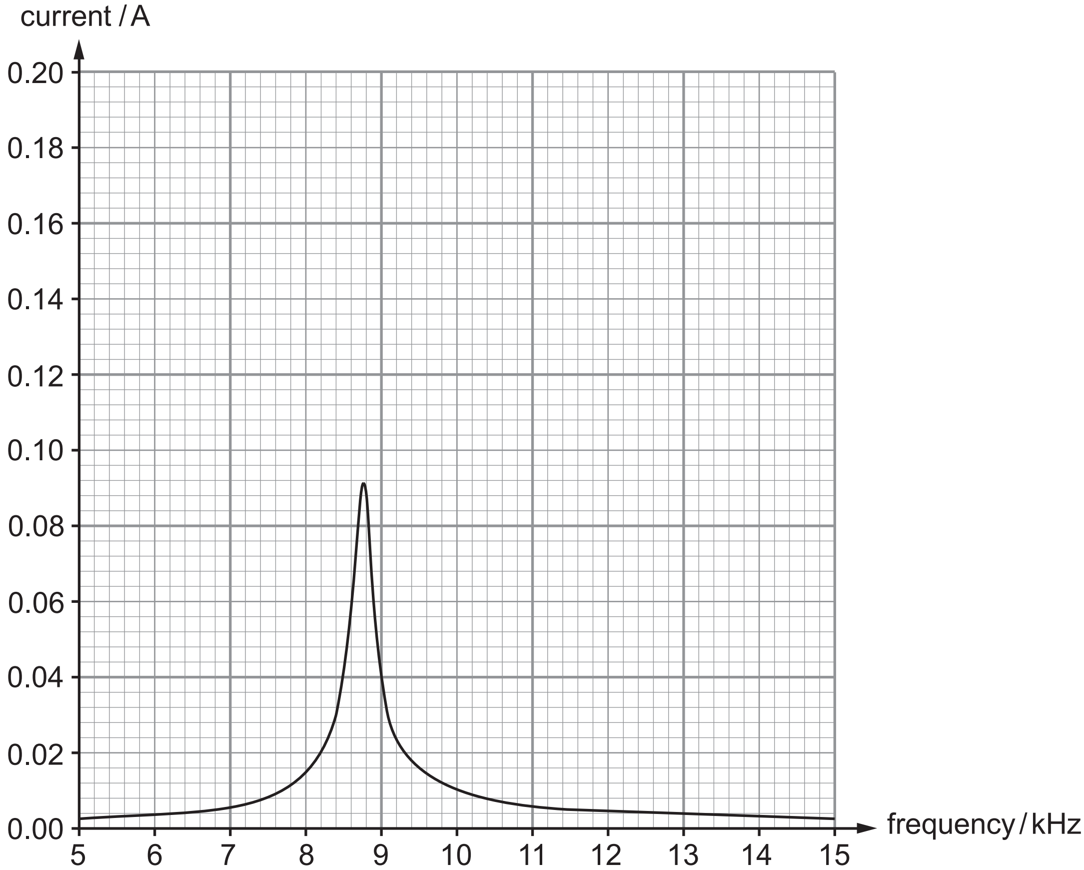
(c) The resonance curve for the circuit with R = 35 Ω and C = 15 nF is shown below.



(i) Add **two labelled** curves to the graph showing the variation of current against frequency when R = 20 Ω and C = 15 nF and when R = 50 Ω and C = 15 nF. [4]
Space for your calculations if needed.

Examiner only

(ii) Use the equation $Z = \sqrt{\left(\omega L - \frac{1}{\omega C}\right)^2 + R^2}$ to explain in detail why the current varies with frequency as shown in the graph (no calculations are required). [6]

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Examiner only

Option B: Revolutions in Physics – The Newtonian Revolution

9. *(a)* (i) Tycho Brahe and another astronomer, 570 km away, were able to make simultaneous angle measurements on heavenly bodies. They could measure parallax down to about $\frac{1}{60}$ degree (0.017°) due to their different locations. Show clearly, **giving a labelled diagram**, that the furthest distance away of a body for which they could measure parallax was about 2×10^6 km. Assume the body to be directly overhead. [3]

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(ii) Tycho Brahe and his associate could easily measure parallax for the Moon (0.40×10^6 km away) but were barely able to detect the parallax of a comet which appeared in 1577. Explain why this provided evidence against Aristotle's division of the universe into sublunary and superlunary (beyond the Moon) regions, where different laws applied. [2]

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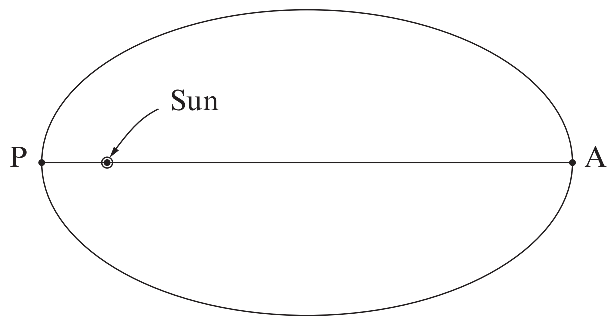
Examiner only

(b) (i) A planet's speed and distance from the Sun at perihelion (P) and aphelion (A) are related by:

$$r_P \, v_P = r_A \, v_A$$

Derive this from Kepler's *equal area* law (Kepler's second law), **adding to the diagram** to assist your explanation. [3]



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(ii) For Jupiter, $r_P = 7.41 \times 10^8$ km and $r_A = 8.16 \times 10^8$ km. Evaluate the **percentage** change in Jupiter's speed as it goes from A to P. [2]

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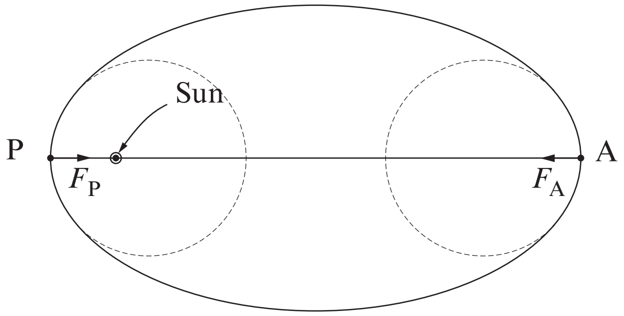
Examiner only

(iii) At P and A a planet's elliptical path coincides with circles of equal radius.



I. Show that the ratio of forces $\frac{F_P}{F_A}$ on the planet at P and A is $\frac{v_P^2}{v_A^2}$. [1]

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II. Use the relationship in *(b)*(i) to show clearly that this is consistent with an inverse square law force between the Sun and planet. [1]

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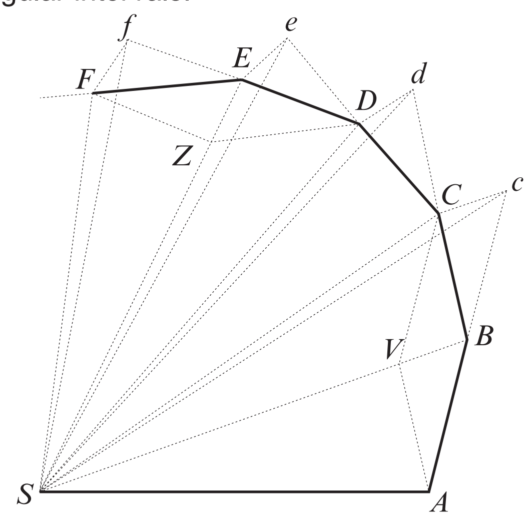
(c) The diagram, from Newton's *Principia,* shows the motion of a body acted upon by short, sharp forces at regular intervals.



(i) In which direction(s) do these forces act? [1]

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Turn over.

Examiner only

(ii) State what Newton was able to show about the triangles ASB, BSC, CSD and how the argument could be applied to the motion of planets. [3]

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(iii) Over forty years before Newton published the *Principia,* Descartes had proposed a quite different explanation for why planets orbited the Sun. What was Descartes' explanation, why did many people find it more satisfying than Newton's, and why is it now almost forgotten? [4]

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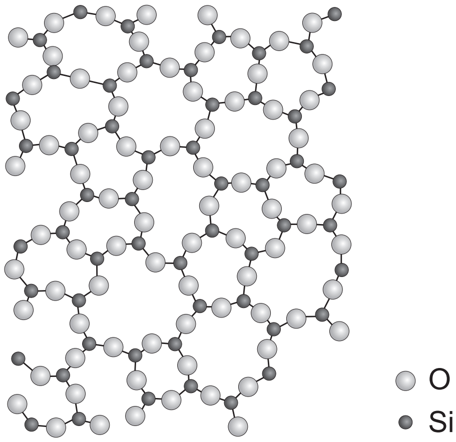
Examiner only

Option C: Materials

10. *(a)* The picture shows the microscopic structure of glass, an amorphous solid.



Explain the following macroscopic properties of glass.

(i) Glass fibres are brittle, showing no plastic deformation before fracture. [2]

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(ii) A sheet of glass can be fractured accurately and cleanly if its surface is scratched and then the glass is bent slightly. [You may wish to draw a labelled diagram to support your answer.] [2]

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Turn over.

Examiner only

(iii) During production, car windscreens are strengthened by using jets of cold air to cool the outer surfaces of the hot glass. This causes the outside to contract quickly while the inside remains soft. Later, the inside cools and contracts. Explain how this process makes the windscreen difficult to break. [2]

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(b) Advanced aircraft and spacecraft applications require aluminium based alloys which are able to operate under extremely diverse conditions. The properties of three of these alloys are given in the table.

Aluminium alloy	Young modulus / GPa	Yield strength / MPa	Maximum tensile strain / %
Alloy A	80	800	9
Alloy B	80	1 000	5
Alloy C	60	600	15

The graph opposite shows the stress against strain graph for **alloy A**.

(i) Use information from the graph to confirm that the Young modulus of alloy A is 80 GPa. [1]

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(ii) Alloy A is in the form of a cylinder of length 2.5 m and diameter 2.5 mm. Determine the work done to stretch this alloy to breaking point. [3]

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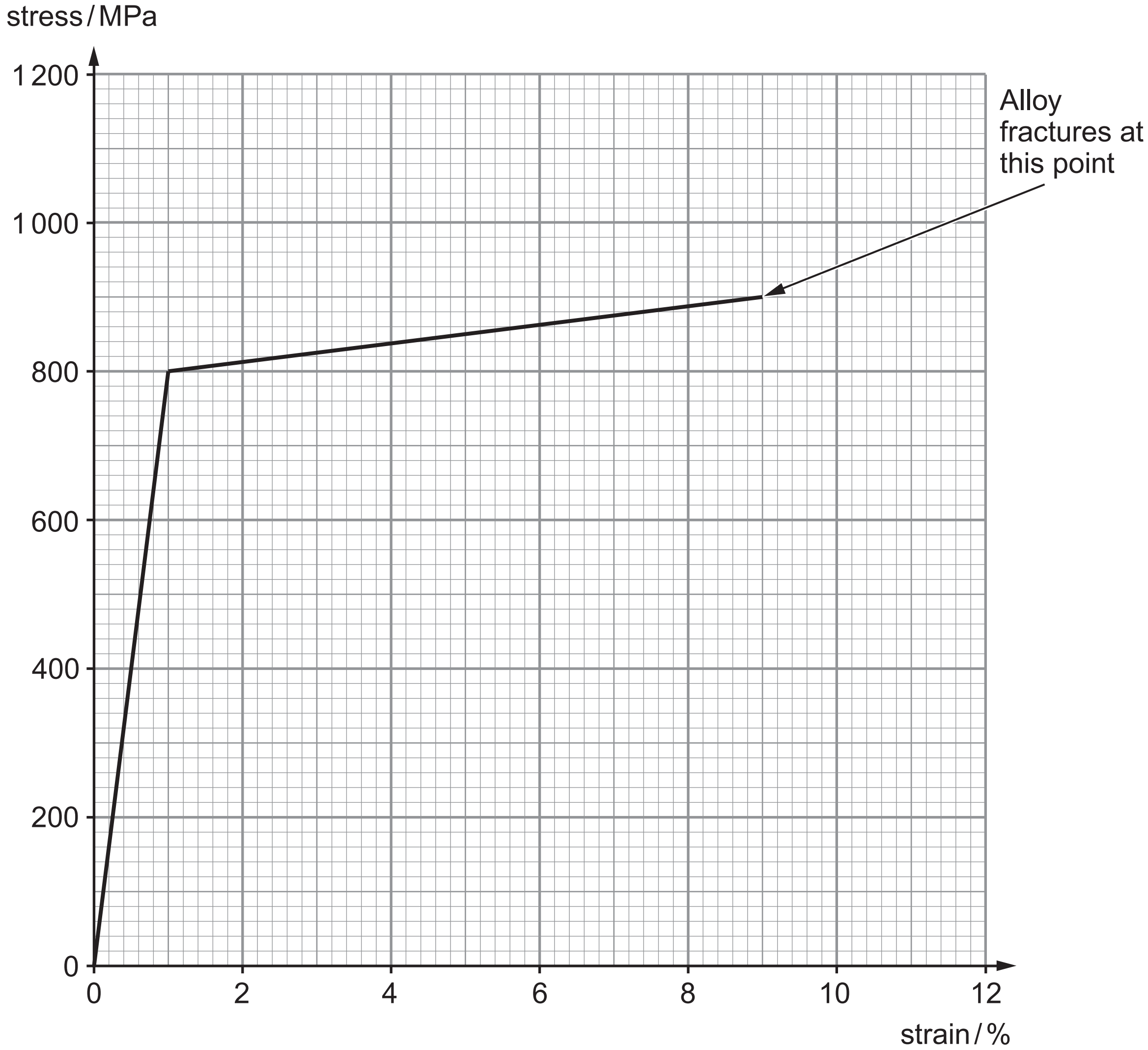
Examiner only



(iii) Draw, on the same axes, a simplified stress against strain graph for **alloy B** using data from the table opposite. [3]

Examiner only

(iv) When these alloys are placed under a constant stress over a long period of time all three undergo *creep* leading to *necking* and eventually fracture.

I. Explain the terms in italics. [2]

Creep: ....

....

Necking: ....

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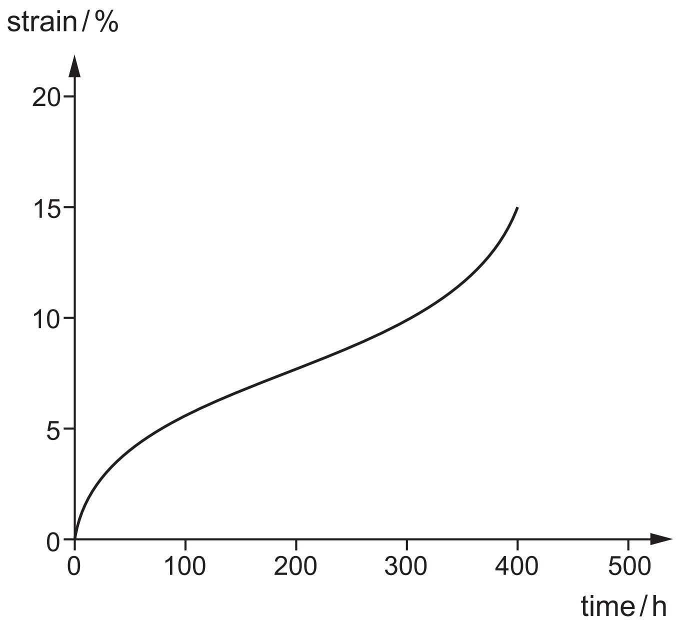
II. A creep curve for **alloy C** is shown. The alloy was subjected to a constant stress of 100 MPa. Using information in the table on page 28, sketch on the same graph (below) a creep curve which could represent alloy C when it is subjected to a constant stress of 120 MPa. [2]



(v) During production the strength of the alloy is controlled by a process called cold working (or work hardening). Describe this process and explain how it increases the strength of the alloy under production. [3]

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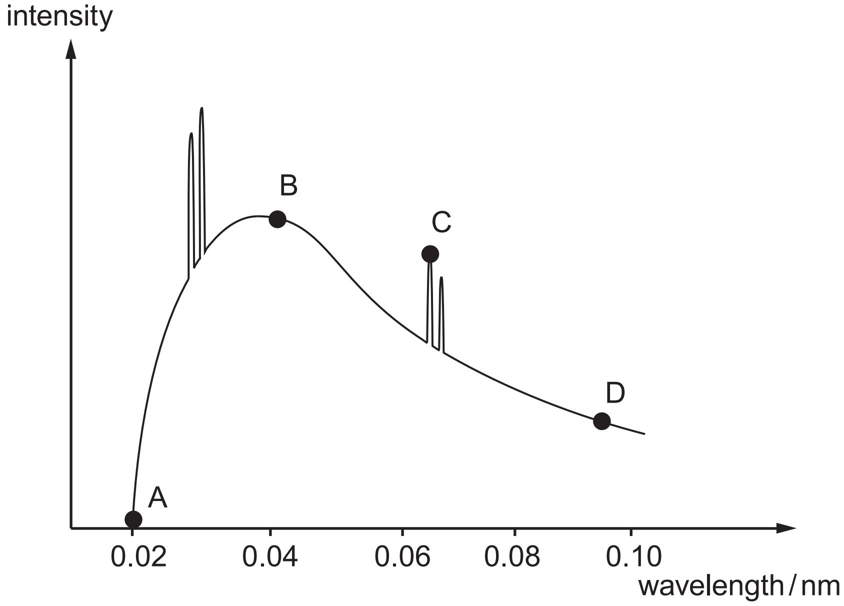
Examiner only

Option D: Biological Measurement and Medical Imaging

11. An X-ray spectrum is shown.



(a) Which of the above points correspond to X-ray photons produced:

(i) by rapidly decelerating electrons; .... [1]

(ii) after knocking out the inner electrons of the target atom; .... [1]

(iii) with the greatest energy per photon? .... [1]

(b) Use information from the graph to calculate the accelerating pd used in the X-ray tube. [2]

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(c) Before taking an X-ray photograph the X-ray beam emerging from the tube is passed through an aluminium filter. Explain why it is necessary for the X-rays to be filtered. [2]

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  Turn over.

Examiner only

(d) (i) Name the region of the electromagnetic spectrum used in MRI scans. [1]

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(ii) Briefly explain the function of this electromagnetic radiation in the working of an MRI scan. [2]

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(iii) Describe one disadvantage of MRI scanning as an imaging technique. [1]

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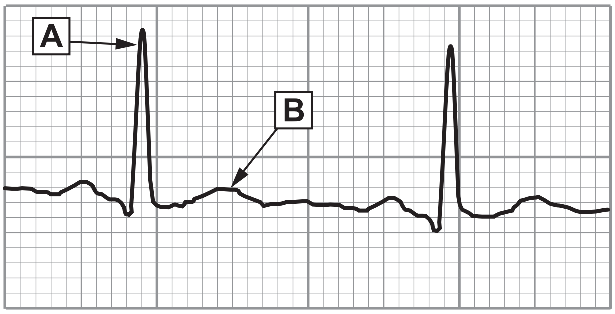
Examiner only

(e) The following diagram shows an ECG trace for a healthy person.



State what electrical event and physical change occur at:

(i) Point A; [2]

Electrical event

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Physical change

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(ii) Point B. [2]

Electrical event

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Physical change

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Turn over.

Examiner only

(f) Ultrasound can be used to measure the speed at which blood is flowing. When reflected off a red blood cell, the wavelength of the ultrasound changes.

(i) What is the name given to this effect? [1]

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(ii) If ultrasound of wavelength 500 μm is used, its speed when travelling through blood is 1 500 m s^{-1} and the wavelength received at the detector is 500.4 μm. Calculate the speed of the flow of blood. [2]

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(g) (i) PET scans are often used to detect tumours. What part of the electromagnetic spectrum do PET scanners detect? [1]

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(ii) Why are PET scanners **not** commonly used in district hospitals? [1]

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Examiner only

Option E: Energy Matters

12. *(a)* Explain why it is important to enrich uranium before it is suitable to be used in a fission nuclear power station. [3]

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(b) In a breeder nuclear reactor uranium-238 is changed into plutonium. Explain the advantage of this and how it is achieved. [2]

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(c) State **two** possible advantages of deuterium-tritium fusion over uranium-235 fission. [2]

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Examiner only

(d) For nuclear fusion to be a viable energy resource a deuterium-tritium plasma must have a large enough confinement time (τ), a high enough temperature (T) and a high enough concentration (number per m^3) of deuterium and tritium particles (n). These conditions are usually expressed as:

$$\tau T n \geq 3.5 \times 10^{28}\ \text{s K m}^{-3}$$

(i) Explain why a high temperature (T) is necessary. [3]

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(ii) A confinement time (τ) of 0.9 s and a temperature of 120 million Kelvin are attainable. Calculate the minimum density of plasma required in kg m^{-3} (the mean mass of deuterium and tritium ions is 2.5 u). [3]

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(e) The energy that can be produced from 1 kg of uranium-235 is 8.3×10^{13} J whereas the energy available from 1 kg of deuterium-tritium is 3.4×10^{14} J. Calculate the energy that can be produced from 1 kg of anti-matter (remember that anti-matter and matter annihilate). [2]

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Examiner only

(f) Coal, biomass, uranium-235, natural gas and wind are five energy resources with similar mean costs per MWh of energy production (£40-£60 per MWh). Discuss other advantages **and** disadvantages of all five energy resources. [5]

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GCE A level

1325/01-B

S15-1325-01B

PHYSICS – PH5
ASSESSMENT UNIT

A.M. THURSDAY, 18 June 2015

CASE STUDY FOR USE WITH SECTION B

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Paragraph

Rocket Science without the Chemistry

Diagram 1

Introduction

It's quite remarkable how much analysis of rocket
motion can be done when one is armed with some 1
physics, a bit of mathematics and a spread sheet.

A simple rocket system that converts plastic bottles
to rockets can be bought relatively cheaply. A good
example is the aquapod®, upon which the system
in the photograph is based. It's an upside-down,
pressurised plastic bottle, half filled with water. When
released, water is ejected at high speed from its tail 2
end resulting in the rocket accelerating upwards.
Because downward momentum is given to the water,
the rocket gains upward momentum. Early on in the
investigations into these rockets, it was discovered
that a ball placed on top of the bottle was far easier
to investigate and model.

Energy Analysis Considering Gravity

The simplest possible analysis that can be done to the rocket is to apply conservation of
energy. We could approximate that the work done in compressing the gas eventually becomes
gravitational potential energy of the ball. How much energy is stored in the compressed gas?
For this, we need a bit of A level Maths in order to calculate the area below an isothermal
compression in a p-V diagram. This is the result of the integration (that doesn't require learning). 3

$$W = p \times V \times \ln\left(\frac{p}{p_{\text{atm}}}\right) \qquad \textbf{Equation 1}$$

where p is the high pressure inside the bottle, V is the volume of compressed air inside the bottle and p_{atm} is atmospheric pressure.

So a 2 litre bottle half filled with water and with
a pressure of 4.4×10^5 Pa inside it can supply
around 650 J of energy. If all this energy were
transferred to a 0.45 kg ball, the ball should
attain a height of some 150 m. In practice, 4
however, only a small fraction of this energy is
transferred to the ball.

This experiment was carried out with a set of
plastic bottles varying in volume from 500 ml
to 3.0 litre. Each bottle was half full of water
at take-off and each bottle was pumped to 5
a pressure of 4.4×10^5 Pa (from an initial
atmospheric pressure of 1.0×10^5 Pa). The
results are shown in the graph (diagram 2).

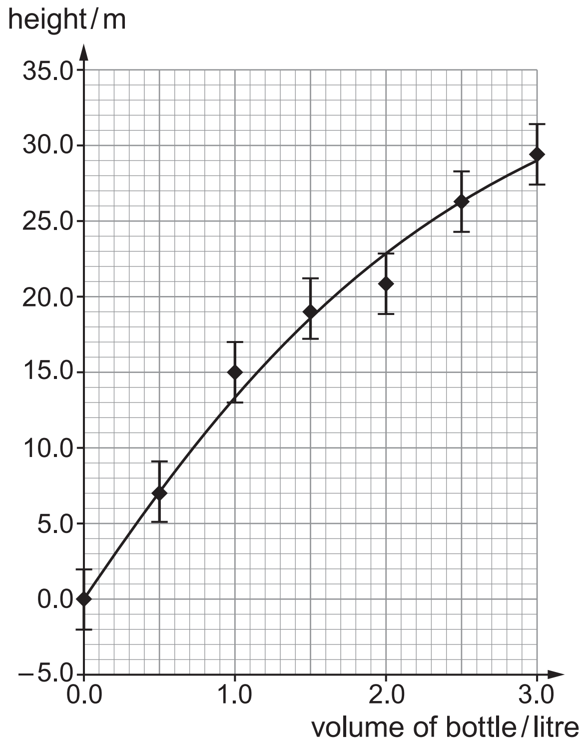

Diagram 2

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Unsurprisingly perhaps, this simplistic conservation of energy argument has not been successful and the relationship between maximum height of the rocket and volume of the bottle is not directly proportional as was predicted by conservation of energy. 6

Ideal Rocket Theory (fixed exhaust speed and ignoring gravity)

Now for some more detailed analysis. We need to look at what causes the acceleration of the rocket. In effect, we need to apply Newton's second law to the rocket and to do that we need to know the rate of change of momentum of water leaving the bottle. First, let's define some terms: 7

m_0 = total initial mass of the rocket;
t = time counting from blast-off;
$\frac{\Delta m}{\Delta t}$ = constant rate of ejecting of mass; 8
u = constant speed of the water leaving the bottle (relative to the rocket).

The resultant force exerted on the water is equal to its rate of change of momentum. The momentum gained by the water per second is $u\frac{\Delta m}{\Delta t}$ (remember, $\frac{\Delta m}{\Delta t}$ is the mass leaving the bottle per second). This, therefore, is the force experienced by the water and by Newton's 3rd law, this is also the force experienced by the rocket. So, we now know that the thrust force acting on the rocket is $u\frac{\Delta m}{\Delta t}$. 9

The mass of the rocket is decreasing at a constant rate of $\frac{\Delta m}{\Delta t}$ so its mass at any time t is given by $\left(m_0 - \frac{\Delta m}{\Delta t}t\right)$. 10

This is enough information to use Newton's 2nd law ($F = ma$) to give an equation for the acceleration. Mathematics (that won't require learning) then leads to solutions for both velocity and height. Here are the equations: 11

$$v = -u\ln(1 - \alpha t) \qquad \textbf{Equation 2}$$
$$h = \frac{u}{\alpha}[(1-\alpha t)\ln(1-\alpha t) + \alpha t] \qquad \textbf{Equation 3}$$

α is the ratio of the rate of loss of mass to the initial mass, $\left(\alpha = \frac{\frac{\Delta m}{\Delta t}}{m_0}\right)$.

We can now try to apply these equations to a typical 2 litre bottle. The total mass of the water, bottle and ball is around 1.5 kg (i.e. m_0 = 1.5 kg). Of this, 1.0 kg is water, 0.45 kg for the ball and the bottle has a mass of 0.05 kg. From high speed video analysis of the rocket, all 1.0 kg of the water is expelled in 0.14 s so that we can calculate the mean rate of decrease of mass of the rocket: 12

$$\frac{\Delta m}{\Delta t} = k = \frac{1.0}{0.14} = 7.1\,\text{kg}\,\text{s}^{-1}$$

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We can also use these figures to determine the exhaust speed of the water when we know that the radius of the bottle neck is 1.1 cm.

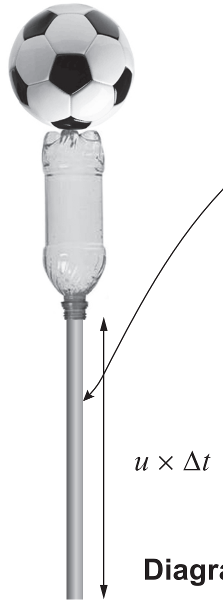

Diagram 3

The mass of the water (Δm) in the cylinder of length $u \times \Delta t$ is:

$$\text{volume} \times \text{density} = \pi r^2 \times u \times \Delta t \times \rho$$

which gives us a relationship between the rate of mass loss and the exhaust velocity:

$$\frac{\Delta m}{\Delta t} = \pi r^2 \rho u \qquad \textbf{Equation 4}$$ 13

When this expression is equated to the actual rate of loss of mass (7.1 kg s^{-1}), it gives a value of u of around 19 m s^{-1} (remembering that the density of water is rather conveniently 1 000 kg m^{-3}).

Comparison Between Ideal Rocket Theory and Experiment

This is enough information to put into the rocket equations (2 & 3) and compare with the motion of an actual rocket. The easiest way of doing this is to use a spreadsheet - we can enter both the rocket equations and actual data and compare the two. In the graph shown (diagram 4), the actual 2 litre rocket data is shown as a continuous line. The theoretical rocket equation is represented by the dotted lines. The data
for the height of the actual rocket was 14
gathered by using a comparatively cheap digital camera set to 220 frames per second. The video of the rocket motion was then analysed frame by frame using a 30 cm ruler to measure distances on the screen and the continuous curve in the graph obtained.

Interestingly, when these data and the rocket equations were put into a spreadsheet, the value of exhaust speed (u) of
19 m s^{-1} did not produce an ideal fit (see 15
diagram 4). The best fit for the early motion of the rocket was provided by an exhaust speed of around 21 m s^{-1} whereas the later motion of the rocket fits better with an exhaust speed of around 18 m s^{-1}. This may seem like a bad agreement but, on the other hand, these discrepancies could be pointing toward the reason for the disagreement.

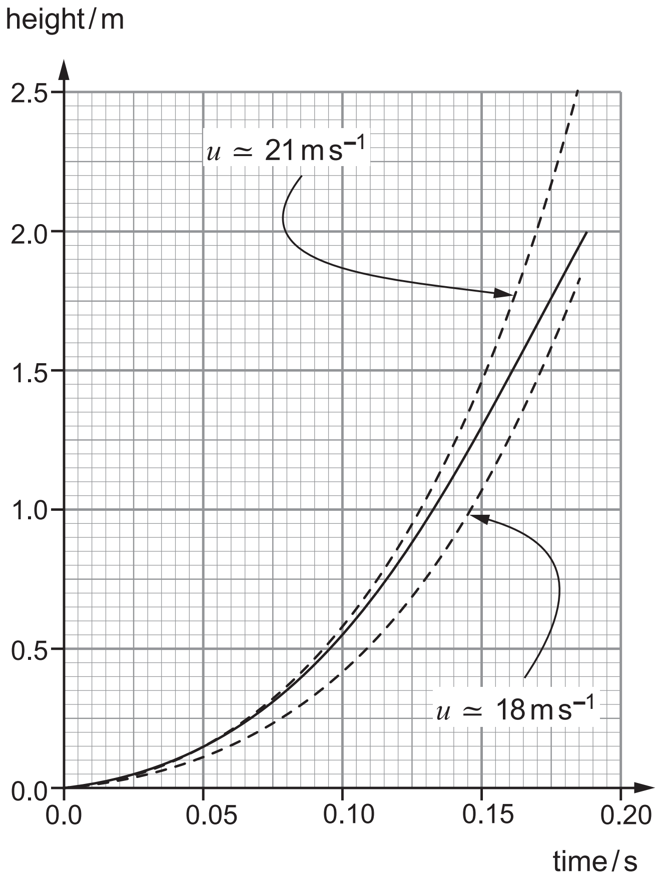

Diagram 4

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Rocket Theory with Decreasing Pressure and Exhaust Speed

The rocket equation fits well for the first 0.10 s of its flight with an exhaust speed of around 21 m s^{-1} but then the actual rocket does not keep up with its theoretical counterpart and the actual rocket seems to fit better with an exhaust speed of around 18 m s^{-1}. What could be the reason for this? Put simply, it's the decrease in pressure of the air inside the bottle as the water is leaving. But how can we model the pressure inside the bottle? Simplistically, we can use Boyle's law. Robert Boyle in the mid 1600s said for a fixed mass of gas at constant temperature: 16

$$\text{pressure} \times \text{volume} = \text{constant}$$

We know that the initial volume of the gas is around 1 litre (for the 2 litre bottle). The final volume of the gas rather obviously will be 2 litre. Boyle's law therefore tells us that the pressure at the start will be approximately double the final pressure. In between these two stages, each gramme of water that is expelled provides an extra 1 cm^3 of air in the bottle and the corresponding pressure drop can easily be calculated using Boyle's law. 17

Now that we have the details to model the pressure drop in the bottle, it is possible to calculate the speed of the water coming out of the bottle. All we have to do is use Bernoulli's equation.

$$p_{\text{atm}} = p - \tfrac{1}{2}\rho u^2 \qquad \textbf{Equation 5}$$

Surprisingly enough, this means that the exit speed of the water is independent of the size of the bottle opening! 18

$$u = \sqrt{\frac{2(p - p_{\text{atm}})}{\rho}} \qquad \textbf{Equation 6}$$

Now we can use this equation to calculate the exhaust speed of the water. The density of water (ρ) is 1 000 kg m^{-3} and the initial ($p - p_{\text{atm}}$) was 3.4×10^5. This gives an initial exhaust speed of around 26 m s^{-1}. 19

For completion, gravity and air resistance should also be incorporated into our model. Gravity is easy enough but what about air resistance? A simple theory for air resistance is that the increase in air resistance is proportional to velocity squared. In fact, if we look up the air resistance of a sphere, we should find: 20

$$F_{\text{drag}} = 0.47 \times \tfrac{1}{2}\rho_{\text{air}} v^2 \times A \qquad \textbf{Equation 7}$$

where A is the maximum cross-sectional area of the sphere, ρ_{air} is the density of air and v is the speed of the sphere.

Another great advantage of placing a football on top of the water bottle rocket is that the air resistance can be modelled based on the dimensions of the football. This assumes that the bottle underneath the football has no effect on the air resistance but should be a reasonable approximation considering that the cross-sectional area of the football is far greater than that of the bottle. The density of air (ρ_{air}) is 1.20 kg m^{-3} and the diameter of the football is 22.0 cm and they can both be inputted into the air resistance equation. 21

Paragraph

All this information should give us a final resultant force acting on the rocket of:

$$F_{\text{res}} = \pi r^2 \rho u^2 - mg - 0.0107v^2 \qquad \textbf{Equation 8}$$

where: u = instantaneous exhaust speed of the water
r = radius of the bottle opening 22
ρ = density of water (1 000 kg m^{-3})
m = instantaneous mass of the rocket (including the water and football)
v = instantaneous speed of the rocket

Final Comparison between Theory and Experiment

When all this data is put into a spreadsheet with time going up in steps of 1/220th of a second (to match the digital camera) and all rocket data calculated for all the time intervals. This is the end result. 23

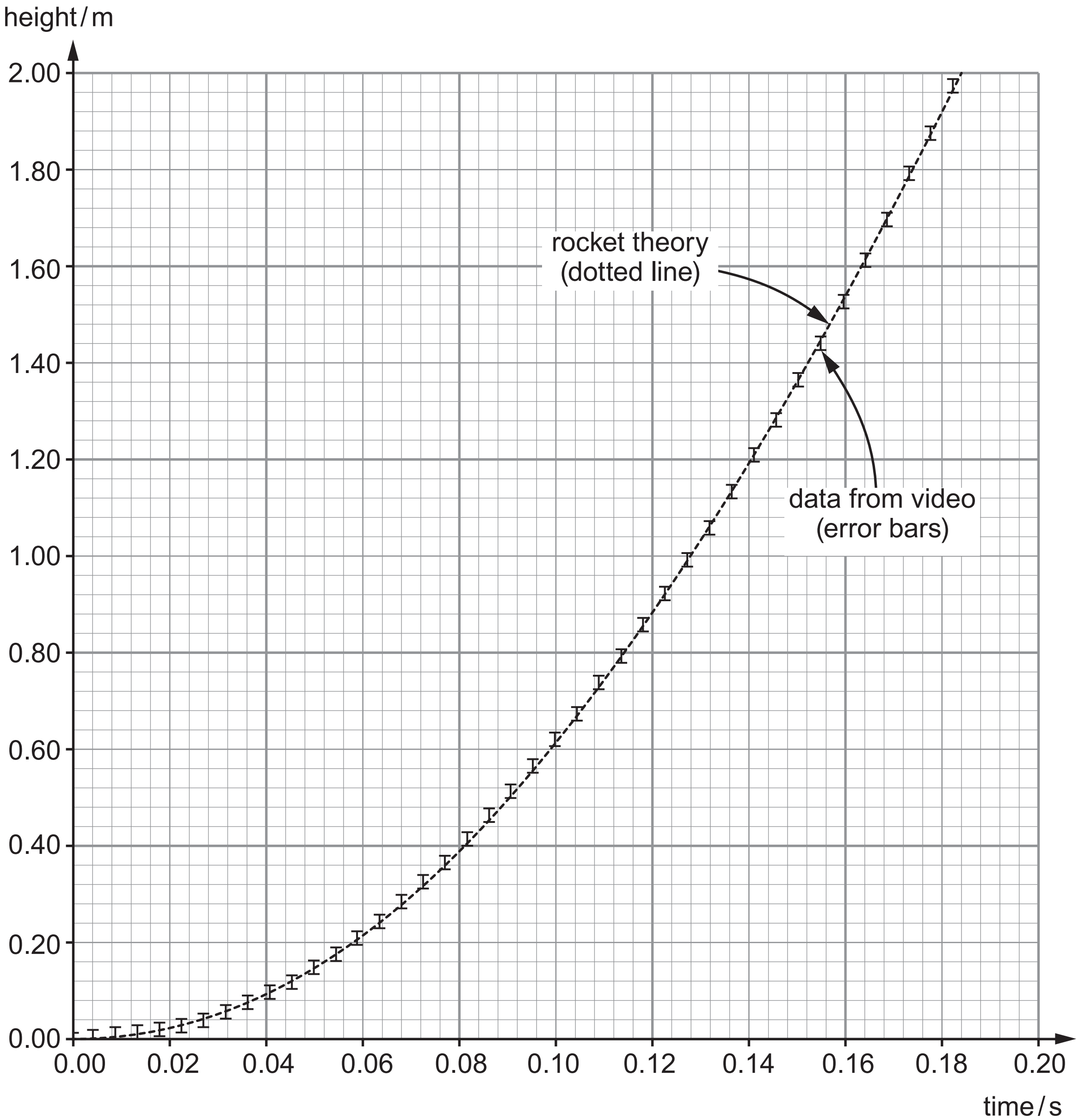

Diagram 5

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Here, the rocket theory (dotted line) is in excellent agreement with the experimental results (error bars). At no point is the computed model data outside the error bars corresponding to the actual motion of the rocket (these error bars simply correspond to ±0.5 mm reading from the ruler next to the computer screen). The final best fit parameters used were m_0 = 1.52 kg, initial pressure = 4.7×10^5 Pa and radius of bottle opening = 1.019 cm. 24

In conclusion, the motion of a plastic water bottle rocket has been analysed using purely A level Physics with a touch of Bernoulli's equation and drag theory. Although the mathematics used can be complicated, this is relatively easily remedied by using numerical methods in a computer spreadsheet. The results are astonishingly accurate and were aided hugely by the novel idea of a football on top of the rocket. 25